Physics of Railgun

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1 Introduction

The anime とある科学の超電磁砲 (romanized as *Toaru Kagaku no Choudenjihou*, translated as *A Certain Scientific Railgun*) has a main character, Mikoto Misaka who is able to use electromagnetic fields to accelerate metal objects to high velocity. However, there are those among us who doubt the physical accuracy of such a power.

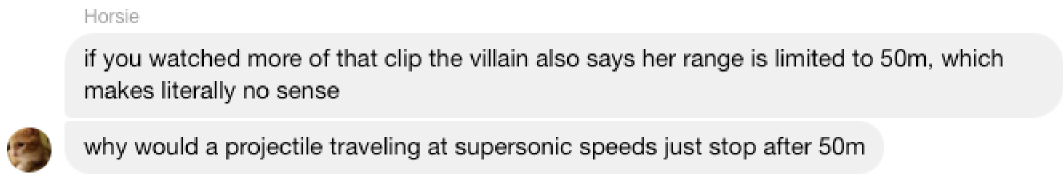

Figure 1: A disbeliever

Thus, we shall begin to analyze her powers with physics.

2 Determining Parameters

First, Misaka usually shoots coins. We will assume the coin she shoots is roughly equivalent to a modern day Japanese 100 yen coin, though the selection of the coin will not vary the end result too much.

A 100 yen coin has a mass of 4.8g, a diameter of 22.6mm, and a thickness or height of 1.7mm. Second, we need to calculate the effect of air resistance on the coin, which is the primary force slowing the motion of the coin as it travels through the air. In order to calculate drag from air resistance, we will need to know ρ, the air density, A, the effective area, and C_d, the coefficient of drag which is a function of the shape of the object as it travels through the air. The density of air at 20° C is 1.2041 kg/m^3.

Figure 2: It is hard to see, but this appears to be a 100 yen coin. (Episode 20, 11:05)

Figure 3: Misaka shooting a coin in the intro (Episode 1, 2:46)

From the image, the face of the coin is clearly visible and is upright, meaning she shoots it such that the face is traveling forward. The coefficient of drag for a flat disk is 1.12. The area of the coin perpendicular to the direction of motion is the area of its face, or πr^2. Thus, $A = \pi r^2 = \pi(\frac{22.6 \text{ mm}}{2})^2 = 4.01 \cdot 10^{-4}$ m^2.

Next, Misaka fires the coins at a velocity of 1030 m/s.

Figure 4: Measured speed of the coin (Episode 1, 7:53)

In order to calculate how long it takes for the coin to hit the ground, we will need to know Misaka's height. She is 161 cm tall, and we can assume her arms are roughly 3/4 of that height, putting the starting height of the coin at 120 cm.

Lastly, we might want to calculate the ballistics for a bullet rather than a coin, which might be interesting. Using the Hornady ELD (extreme low drag) bullets, we find that it has a weight of 13.5g, a diameter of 7.62mm, and a C_d of 0.242. The area is then the area of the face, or πr^2 again. $\pi(\frac{7.62\text{ mm}}{2})^2 = 4.56 \cdot 10^{-5}\text{ m}^2$.

To summarize:

Description	Symbol	Value	Unit
Mass of coin	m	4.8	g
Diameter of coin	d	22.6	mm
Air density	ρ	1.2041	kg/m^3
Area	A	$4.01 \cdot 10^{-4}$	m^2
Coefficient of drag	C_d	1.12	n/a
Initial velocity	v_0	1030	m/s
Time elapsed	t	n/a	s
Height	h	120	cm
Gravity of Earth	g	9.81	m/s^2
Mass of bullet	m	13.5	g
Area of bullet	A	$4.56 \cdot 10^{-5}$	m^2
Coefficient of drag	C_d	0.242	n/a

Table 1: List of variables

3 Problem Statement

Because we are AP physics students, here is a proper problem statement:

Mikoto Misaka, the "railgun", fires a 100 yen coin at 1030 m/s at her latest enemy, who is standing at a distance of 50m away. However, the coin experiences drag from air resistance as it flies through the air with its face towards the enemy.

a. Find the velocity of the coin as a function of time.

b. Find the horizontal distance of the coin from Misaka as a function of time.

c. How far does the coin travel? Does she hit her enemy?

4 Solution

The Drag equation gives the force of drag as $F_d = \frac{1}{2}\rho v^2 AC_d$. For clarity, let $k = \frac{1}{2}\rho AC_d$. The force is also negative since it opposes the velocity of the coin.

$$\begin{aligned}
F_d &= -kv^2 \\
a &= \frac{F}{m} = \frac{\mathrm{d}v}{\mathrm{d}t} \\
\mathrm{d}v &= \frac{F}{m}\,\mathrm{d}t \\
&= -\frac{k}{m}v^2\,\mathrm{d}t \\
\int_{v_0}^{v_f} \frac{1}{v^2}\,\mathrm{d}v &= -\int_0^t \frac{k}{m}\,\mathrm{d}t \\
[-v^{-1}]_{v_0}^{v_f} &= -\frac{k}{m}t \\
\frac{1}{v_0} - \frac{1}{v_f} &= -\frac{k}{m}t \\
v_f &= \frac{1}{\frac{1}{v_0} + \frac{k}{m}t} = \boxed{\frac{v_0}{1 + \frac{v_0 k}{m}t}} \text{ a.}
\end{aligned}$$

Essentially, the velocity starts at v_0 and decreases as time grows bigger like the function 1/x. To find position, simply integrate over velocity. Let $c = \frac{v_0 k}{m} = \frac{v_0 \rho AC_d}{2m}$.

$$\begin{aligned}
x &= \int_0^t v(t)\,\mathrm{d}t \\
&= \int_0^t \frac{v_0}{1+ct}\,\mathrm{d}t \\
&= [\frac{v_0}{c}\ln(1+ct)]_0^t \\
&= \boxed{\frac{v_0}{c}\ln(1+ct)} \text{ b.}
\end{aligned}$$

Lastly, to find the distance traveled we just need to calculate the time it takes before the coin hits the ground (where we then assume it stops moving horizontally).

$$\begin{aligned}
h &= \frac{1}{2}gt^2 \\
t &= \sqrt{\frac{2h}{g}} = 0.5 \text{ s}
\end{aligned}$$

Evaluating $x(t)$ at $t = 0.5$: First, $c = 58.0 \text{ s}^{-1}$, thus $x(t) = \boxed{60.4\text{m}}$.

5 Problem 2

Suppose Misaka is tired of using coins, and upgrades her ammunition to the bullets described above. Assuming the coin reaches terminal velocity from the railgun launch, how far does the bullet travel?

6 Solution

The acceleration of a railgun is given by

$$a = \frac{L'I^2}{2m}$$

where L' is the inductance per meter, and I is the current.

The terminal velocity can be derived from the drag equation given earlier, and is

$$v_t = \sqrt{\frac{2ma}{\rho A C_d}}$$

Combining the two equations yields the terminal velocity for a railgun:

$$v_t = \sqrt{\frac{2ma}{\rho A C_d}} = \sqrt{\frac{2m\frac{L'I^2}{2m}}{\rho A C_d}} = \sqrt{\frac{L'I^2}{\rho A C_d}}$$

Squaring both sides,

$$L'I^2 = {v_t}^2 \rho A C_d$$

Evaluating for the disk A and C_d and v_0 for v_t one obtains 574 $H \cdot$ A^2. Finding $L'I^2$ for a military railgun, $79.0 \cdot 10^4$, or about 138x more powerful which is reasonable considering the substantially smaller payload.

Finding the launch velocity for a bullet with the same terminal velocity equation, but with the bullet A and C_d yields a initial velocity of 6,570 m/s. Finally, c is 3.23 s^{-1} and the distance traveled is 1950m, or about two kilometers. This is an absurd 33x farther distance than the coin, and intuitively a bullet is a much more aerodynamic shape.

7 Conclusion

Railgun physics is absolutely correct; Mikoto Misaka best girl.